# Unit 11: Polynomial functions
# Unit 9 (part of): Transformations

# Study guide: Part I/II

This is part I (of II). The second part is a practice test.
The purpose of this guide is to help you organize (at least conceptually) the material we covered this unit.

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| The test will be WITHOUT the use of a graphing calculator.<br>A simple 4-operations calculator is allowed. |

## Classwork

In these units we used many packets. A small picture of the front page for each packet is attached at the end of this document. If you are missing any of the packets, please look on schoology or come and ask me (I have some copies left, so we can save trees). On schoology, these are all in the assignment called "Packets".
If you have the packet but it is not fully solved, or you are not sure about any part of your solution, please come and ask me (or message me).

## Keywords and terms in these units

The list below includes terms you need to know and understand (in context) from the current units. You are also expected, as usual, to know the material covered so far in the year.

<u>Unit 11: Polynomials  (Chapter 11, Pages 479-513)</u>
Term, coefficient, degree of a term, degree of polynomial, Leading coefficient
Constant, Liner, Quadratic, Cubic
Monomial, Binomial, Trinomial
Roots, zeros
Polynomial of degree 'n' has 'n' zeros
Polynomial of degree 'n' can be factored into 'n' linear factors
Multiplicity of factors
Complex roots come in conjugate pairs      (<-- polynomial with real coefficients)
Division by $(x - x_1)$, where $x_1$ is a root, leaves no remainder

Remainder theorem
Rational roots theorem: for polynomial with integer coefficients  (page 496)
Descartes' rule of signs: Positive real roots related to variations of sign (page 501)
Using Descartes' rule for number of negative real roots (P(-x)).

You need to be proficient with:
Synthetic division
Regular polynomial division

Graphing
End behavior: determined by order of polynomial and sign of leading coefficient
Real roots represent x intercepts
Linear factors of multiplicity 1 represent line crossing the x-axis
Linear factors of multiplicity 2 represent parabola touching the x-axis
Complex roots do not represent x-axis crossing
There are no additional x-axis intercepts to these indicated by the real roots

Unit 9: Transformations (Chapter 9, 9-1 to 9-3, pages 384-399)
Symmetry (We focused with respect to a vertical line. E.g., x=3)
Odd function
Even function
Parent functions: Linear (x), Quadratic (x^2), Cubic (x^3), Absolute Value (|x|) , Radical (sqrt(x)), Rational (1/x), Floor ( floor(x) or int(x) )
Transformations: f(x)+3, f(x)-3, f(x+3), f(x-3), 2f(x), 1/2 f(x), f(2x), f(x/2), f(-x), -f(x)
( A little bit harder transformation, but worth contemplating: f(3x-1) )
Shift/translate : f(x-3), f(x) -3
Stretch/shrink : 2f(x), f(2x)
Reflection (f(-x), -f(x))
Rigid transformation and non-rigid transformations

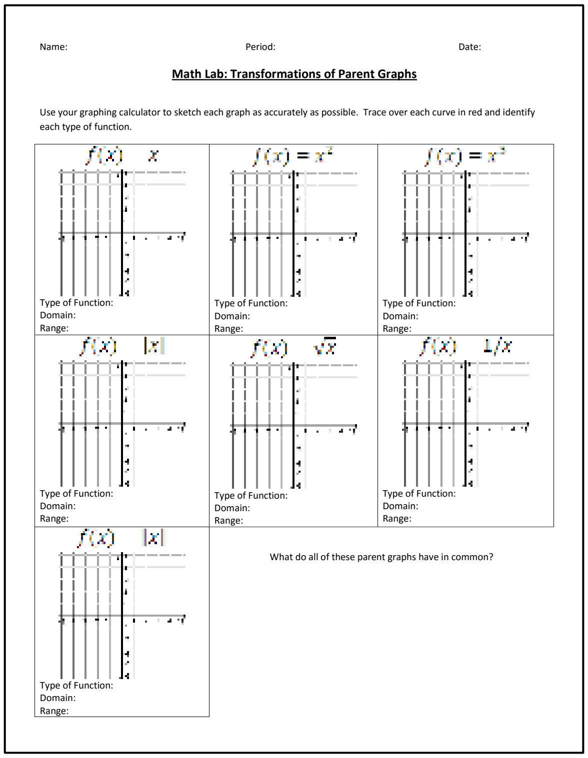
Important to understand for this unit as well:
Function composition: f(g(x))

Review
Quadratics: Standard, Vertex, and factor forms
Linear lines: Equation, Line through point, perpendicular lines, intercepts
Optimization (min/max) problems using quadratics.

## Pictures of the FRONT pages of the packets

The full packets are available on schoology. See an assignment for the test day titled Packages.
(see next page)

Function transformation packet:



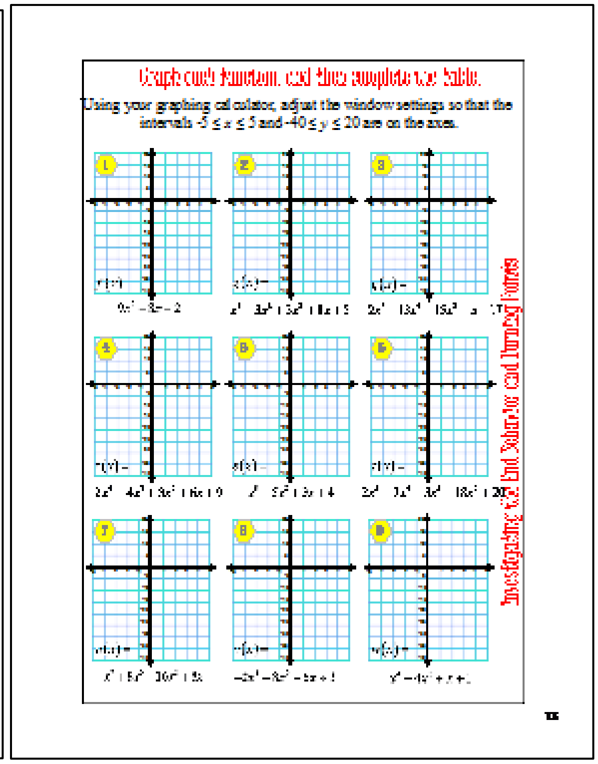
Name: Period: Date:

**Math Lab: Transformations of Parent Graphs**

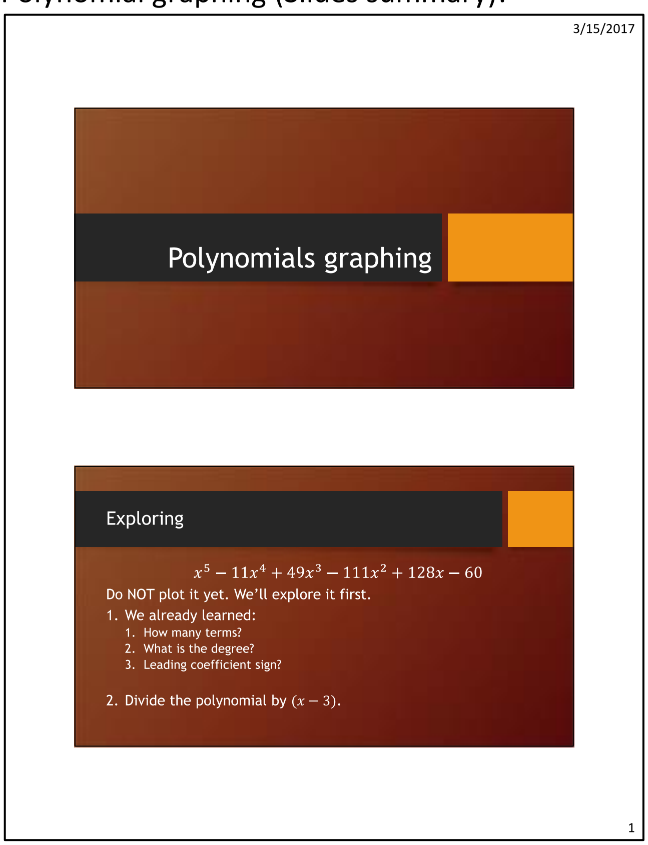
Use your graphing calculator to sketch each graph as accurately as possible. Trace over each curve in red and identify each type of function.

| | | |
|---|---|---|
| $f(x)$ $x$<br>Type of Function:<br>Domain:<br>Range: | $f(x) = x^2$<br>Type of Function:<br>Domain:<br>Range: | $f(x) = x^3$<br>Type of Function:<br>Domain:<br>Range: |
| $f(x)$ $\lvert x \rvert$<br>Type of Function:<br>Domain:<br>Range: | $f(x)$ $\sqrt{x}$<br>Type of Function:<br>Domain:<br>Range: | $f(x)$ $1/x$<br>Type of Function:<br>Domain:<br>Range: |
| $f(x)$ $\lvert x \rvert$<br>Type of Function:<br>Domain:<br>Range: | What do all of these parent graphs have in common? | |

End Behavior packet:




Polynomial graphing (Slides summary):



Polynomial graphing exploration: You got ONE of these two packets (I did put BOTH on schoology):

Exploration in Polynomials graphing

Given the polynomial:

$$P(x) = x^8 - 10x^7 + 47x^6 - 120x^5 + 135x^4 - 10x^3 - 67x^2 + 100x - 156$$

1. How many terms are there in P(x)?
2. What is the degree of the polynomial?
3. What is the sign of the leading coefficient?

You can already determine the end-behavior of the graph.

==

Given that the polynomial has roots at $x = 3$, at $x = (2+3i)$, at $(x = 2)$ it has a root with multiplicity 2, and a root at $x = i$, find all the remaining roots, and factor P(x) to it's linear or quadratic components.

==

Use the space below (and back) for computations, and summarize your results on the next page.

1

Exploration in Polynomials graphing

Given the polynomial:

$$P(x) = x^6 - 6x^5 + 10x^4 - 2x^3 - 3x^2 + 4x - 12$$

1. How many terms are there in P(x)?
2. What is the degree of the polynomial?
3. What is the sign of the leading coefficient?

You can already determine the end-behavior of the graph.

==

Given that the polynomial has roots at $x = 3$, at $x = 2$ it has a root with multiplicity 2, and a root at $x = i$, find all the remaining roots, and factor P(x) to it's linear or quadratic components.

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Use the space below (and back) for computations, and summarize your results on the next page.

1

Describing polynomials:

Name: Period: Date:

**Practice Worksheet: Describing Polynomials**

1. An _____________ degree polynomial must have at least one real zero.
2. A polynomial function is written in ____________ _____________ if its terms are written in descending order of exponents from left to right.
3. The _____________ _________________ is the number in front of the term with the highest exponent in the polynomial.
4. A _________________ is a polynomial with one term, a ___________________ has two terms, and a _______________ has three terms.
5. It is possible for an _____________ degree polynomial to have no real zeros.
6. The _____________ _____________ ___________ is used to determine the end behavior of the graph of a polynomial function.

**Write each polynomial in standard form and state the degree, type, leading coefficient, and draw arrows indicating the end behavior. The first example has been done for you.**

| | Standard Form | Degree | Classify by degree | Classify by number of terms | LC | End Behavior |
|---|---|---|---|---|---|---|
| *Example:* $y = 7 - 2x$ | $y = -2x+7$ | 1 | linear | binomial | -2 | |
| 7. $y = 2x - x^3 + 8$ | | | | | | |
| 8. $y = 3x^2 + x^3 - (x^3 + x^2)$ | | | | | | |
| 9. $y = (2x)^3 + 3x - 1$ | | | | | | |
| 10. $y = (x+2)^2 + 3$ | | | | | | |
| 11. $y = (2+x)(2-x) - 4$ | | | | | | |
| 12. $y = 3(x+1)^2 - 3x^2$ | | | | | | |
| 13. $y = 2x - 2(x-3)$ | | | | | | |

Descartes' rule:

Descartes' Rule of signs

**Question 1:**

| P(x) | Sign Variations | Possible Positive roots | Actual positive roots | Sign variations of P(-x) | Possible negative roots | Actual negative roots |
|---|---|---|---|---|---|---|
| $x^2 - x + 1$ | | | | | | |
| $x^2 - 4x + 1$ | | | | | | |
| $x^2 - 2x + 1$ | | | | | | |
| $x^2 - \_?\_x + 1$ | | | | | 1 | |
| $x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$ | | | | | | |
| $x^6 - x^5 + x^4 - x^3 + x^2 - x + 0$ | | | | | | |
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